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SLOW TWO-PHASE FLOW THROUGH A SINUSOIDAL CHANNEL

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Abstract—The two-phase flow through a symmetric sinusoidal channel is studied by means of a regular perturbation analysis, where the small parameter is defined as the ratio between the amplitude of variation of the channel wall and the average thickness of the non-wetting phase. Results are valid for Reynolds numbers of the same order of magnitude as that of the expansion parameter. It is thus found that the fluid-fluid interface presents a wavy shape characterized by an amplitude and a phase-shift with respect to the fixed solid-fluid interface. Instabilities of the two-phase flow can arise for large values of the viscosity, flow rate and phase thickness ratios. Results are expected to be a first step towards the understanding of the hydrodynamics of trickle bed reactors, where several flow regimes are possible.

INTRODUCTION

Two-phase flows through interconnected conduits and channels are present in much equipment of industrial operations and processes, and for design one needs, therefore, to predict the behaviour of bulk phases and interfaces under different physicochemical situations and operating conditions.

One of the most relevant examples of such equipment is the trickle-bed reactor, where liquid and gas phases are made to flow through the interconnected conduits of a packed bed, so that a heterogeneous chemical reaction takes place on the catalytic solid bed. Nevertheless, the flow regime in this system is not necessarily unique, since different types of flows can be observed depending upon the relation between the liquid and gas flow rates, amongst other important parameters.

Several studies of two-phase flows through conduits of simple geometries have been performed (see, for example, Dukler 1977; Hickox 1971; Esmail *et al.* 1975; Ardron 1980). However, these results are only an idealized representation of two-phase flows in more complex systems, such as the trickle-bed reactor or natural porous media, where conduits have an axially varying cross-sectional area of flow and the Lagrangean acceleration is present (Deiber & Schowalter 1981). Although the flow of one fluid through peridically constricted tubes has been solved for different purposes (Chow & Soda 1972; Azzam & Dullien 1976; Fedkew & Newman 1977; Deiber & Schowalter 1978) the two-phase flow in this type of geometry has not been studied yet, despite its direct connection with technological interests and needs.

This work presents an analysis of the two-phase flow through a sinusoidal channel, where the magnitude of variation in the cross-sectional area of flow is assumed small in relation to the average thickness of the non-wetting phase. Therefore, as a step toward the hydrodynamic modelling of a trickle-bed reactor, a regular perturbation analysis is performed to evaluate the velocity field of the stratified two-phase flow and its interfacial shape. One thus finds that the phase shift γ and amplitude Am depend on the capillary number, physical and geometrical parameters.

EQUATIONS OF THE FLOW

Two Newtonian, incompressible and immiscible phase flow in steady state through a symmetric sinusoidal channel, which is infinitely wide in the direction normal to the x–y plane

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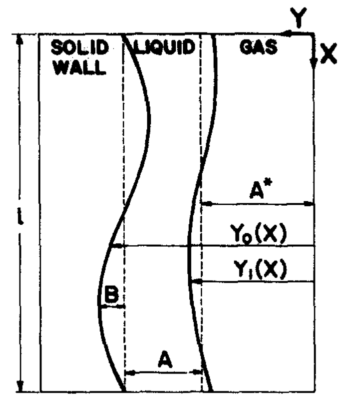

Figure 1. Two-phase flow through a sinusoidal channel. In this case, the wetting phase is a liquid, and the non-wetting phase is a gas.

(figure 1). Therefore, A refers to the average thickness of the fluid phase in contact with the solid wall, and A^* is the average thickness of the other phase. Since the channel wall is periodic, the solid-fluid interface is expressed,

$$Y_0(x) = 1 + m + \epsilon \sin \lambda x \qquad [1]$$

and, similarly, the fluid–fluid interface is,

$$Y_1(x) = 1 + f(x) \qquad [2]$$

where $m = A/A^*$, $\epsilon = B/A^*$ and $\lambda = 2\pi A^*/l$. In these relations, l is the wave length of the periodic variations of the channel wall, and B is the magnitude of variation in the cross area of flow. In [2], $f(x)$ is unknown, and must be determined with the problem solution.

For the wetting phase, the dimensionless components of the Navier–Stokes equation and the continuity equation can be written,

$$\mathrm{Re}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + N \qquad [3]$$

$$\mathrm{Re}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \qquad [4]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad [5]$$

where $N = gA^{*3}/\nu Q$, with g the gravity acceleration only present in the x-direction. Similar equations can be formulated for the other phase.

Consequently, the use of the stream function yields,

$$\mathrm{Re}\,(\psi_y \nabla^2 \psi_x - \psi_x \nabla^2 \psi_y) = \nabla^4 \Psi \qquad [6]$$

$$\mathrm{Re}\,(\psi_y^* \nabla^2 \psi_x^* - \psi_x^* \nabla^2 \psi_y^*) = \frac{\alpha}{\theta} \nabla^4 \psi^* \qquad [7]$$

where $\mathrm{Re} = Q/\nu$ is the Reynolds number, Q is the axial flow rate per unit length perpendicular to the flow plane, $\alpha = \mu^*/\mu$ is the ratio of viscosities and $\theta = \rho^*/\rho$ is the relation between densities. A star as superscript in any sumbol is used to designate the phase that does not wet the channel wall. In [6] and [7], the stream functions have been defined so that the axial velocity

u and its normal velocity v can be written as follows:

$$u = \psi_y \quad , \quad v = -\psi_x \qquad [8]$$

and, similarly,

$$u^* = \psi_y^* \quad , \quad v^* = -\psi_x^* \qquad [9]$$

where the velocity scale is Q/A^*, and the length scale is A^*.

The boundaries of the flow domain are irregular in relation to the Cartesian coordinate system (figure 1) and one is unknown; hence, it is convenient to introduce, in dimensionless form, the normal $n(x)$ and tangential $t(x)$ coordinates to the boundaries described by [1] and [2]. Consequently, since fluids are not allowed to slip at the wall,

$$y = Y_0(x) \quad , \quad \frac{\partial \psi}{\partial n} = 0. \qquad [10]$$

On physical grounds, symmetry is assumed at the channel center,

$$y = 0 \quad , \quad \frac{\partial^2 \psi^*}{\partial y^2} = 0. \qquad [11]$$

In addition, at the fluid-fluid interface the following kinematic conditions are imposed,

$$y = Y_1(x) \quad , \quad \begin{array}{ll} \psi = \psi^* = 0 & [12] \\ \dfrac{\partial \psi}{\partial n} = \dfrac{\partial \psi^*}{\partial n} & [13] \end{array}$$

while, the stresses $\mathbf{T}$ and $\mathbf{T}^*$ must satisfy,

$$y = Y_1(x) \quad , \quad \begin{array}{ll} T_{nn}^* - T_{nn} = 2\,H\,\Lambda^{-1} & [14] \\ T_{nt}^* = T_{nt} & [15] \end{array}$$

where $\Lambda = \mu Q/\sigma A$ is the capillary number, σ is the fluid–fluid interfacial tension, and the interfacial curvature is expressed,

$$2H = \frac{f_{xx}}{(1+f_x^2)^{3/2}}. \qquad [16]$$

Additional boundary conditions at the wall and at the center line of the channel need to be specified and one can, therefore, use the constraint of flow-rate conservation to obtain,

$$y = 0 \quad , \quad \psi^* = -q \qquad [17]$$

$$y = Y_0(x) \quad , \quad \psi = 1 \qquad [18]$$

where $q = Q^*/Q$ is the relation between flow rates of each phase.

It can be observed that the problem of the two-phase flow through a periodically constricted channel involves [6]–[18], and despite the unknown velocity fields, $Y_1(x)$ must be obtained as a part of the solution.

A REGULAR PERTURBATION SOLUTION

It is clear from the above equations, that an exact analytic solution of the formulated problem is very difficult to obtain, and a numerical algorithm must be used; a step under development. In this work, however, a regular perturbation analysis is performed with ϵ as small parameter and requiring $\lambda \sim 0(1)$ and $\mathrm{Re} \sim 0(\epsilon)$. Thus, the approximate solution is valid for small Reynolds numbers and small values of the relation between the magnitude of the variation in the cross area of flow B, to the average thickness of the non-wetting phase A^*. Therefore, the following expansions are proposed,

$$\psi(x, y) = \sum_{s=0}^{\infty} \psi_s \epsilon^s$$

$$\psi^*(x, y) = \sum_{s=0}^{\infty} \psi^*_s \epsilon^s$$

$$p(x, y) = \sum_{s=0}^{\infty} p_s \epsilon^s \qquad [19]$$

$$p^*(x, y) = \sum_{s=0}^{\infty} p^*_s \epsilon^s$$

$$f(x) = \sum_{s=1}^{\infty} g_s \epsilon^s$$

and the pressure drop measured along the wavelength l is,

$$\Delta p = \sum_{s=0}^{\infty} \Delta p_s \epsilon^s.$$

These equations can also be expanded in Taylor series around $y = 1 + m$ and $y = 1$ in order to avoid the evaluation of the boundary conditions on the irregular interfaces. Then, solutions of the problem may be thus obtained for different orders of the expansion ($s = 0, 1, 2 \ldots$ etc) in terms of the small parameter ϵ.

The 0(1) problem

After collecting terms of 0(1) one obtains,

$$\nabla^4 \psi_0 = 0 \qquad [20]$$

$$\nabla^4 \psi^*_0 = 0 \qquad [21]$$

with the following boundary conditions:

at $y = 1 + m$,

$$\psi_{0y} = 0 \quad , \quad \psi_0 = 1 \qquad [22]$$

at $y = 1$,

$$\psi_0 = 0 \quad , \quad \psi^*_0 = 0 \qquad [23]$$

$$\psi_{0y} = \psi^*_{0y} \qquad [24]$$

$$\frac{p_0}{2} + \psi_{0xy} = \frac{p^*_0}{2} + \psi^*_{0xy}\alpha \qquad [25]$$

$$\psi_{0xx} - \psi_{0yy} + (\psi^*_{0yy} - \psi^*_{0xx})\alpha = 0 \qquad [26]$$

at $y = 0$,

$$\psi^*_{0yy} = 0 \quad , \quad \psi^*_0 = -q \qquad [27]$$

Therefore, from [20] to [27], it can be readily found,

$$\psi_0 = ay^3 + by^2 + cy + d \qquad [28]$$

$$\psi^*_0 = a^*y^3 + b^*y^2 + c^*y + d^* \qquad [29]$$

where constant coefficients are obtained from the following system of linear algebraic equations that result after applying the boundary conditions [22]–[27],

$$3a(1+m)^2 + 2b(1+m) + c = 0 \qquad [30]$$

$$a(1+m)^3 + b(1+m)^2 + c(1+m) + d = 1 \qquad [31]$$

$$a + b + c + d = 0 \qquad [32]$$

$$a^* + b^* + c^* + d^* = 0 \qquad [33]$$

$$3a^* + 2b^* + c^* - 3a - 2b - c = 0 \qquad [34]$$

$$\alpha(6a^* + 2b^*) - (6a + 2b) = 0 \qquad [35]$$

$$d^* = -q \qquad [36]$$

$$b^* = 0. \qquad [37]$$

These equations are solved numerically for each set of given physical parameters. Equation [25] can then be used to show that $p_0 = p^*_0$ at $y = Y_1(x)$; a result consistent with the two-phase flow in straight channels. The pressure gradient of 0(1) can also be obtained, because

$$p_{0x} = p^*_{0x} = 6a + N.$$

The 0(ε) problem

After collecting terms of 0(ε), one also obtains,

$$\nabla^4\psi_1 = 0 \qquad [38]$$

$$\nabla^4\psi^*_1 = 0 \qquad [39]$$

with the following boundary conditions:

at $y = 1 + m$,

$$\psi_{0yy} \sin \lambda x + \psi_{1y} = 0 \qquad [40]$$

$$\psi_{0y} \sin \lambda x + \psi_1 = 0 \qquad [41]$$

at $y = 1$,

$$\psi_{0y} g_1 + \psi_1 = 0 \qquad [42]$$

$$\psi_{0y}^* g_1 + \psi_1^* = 0 \qquad [43]$$

$$\psi_{0yy} g_1 + \psi_{1y} = \psi_{0yy}^* g_1 + \psi_{1y}^* \qquad [44]$$

$$\frac{p_1}{2} + \psi_{1xy} + \psi_{0yy} g_{1x} - \frac{\Lambda^{-1}}{2} g_{1xx} = \frac{p_1^*}{2} + (\psi_{1xy}^* + \psi_{0yy}^* g_{1x})\alpha \qquad [45]$$

$$(\psi_{1yy}^* - \psi_{1xx}^* + \psi_{0yyy}^* g_1)\alpha - \psi_{1yy} + \psi_{1xx} - \psi_{0yyy} g_1 = 0 \qquad [46]$$

at $y = 0$

$$\psi_1^* = 0 \quad , \quad \psi_{1yy}^* = 0. \qquad [47]$$

From [42] and [43], the following condition is found at $y = 1$,

$$\psi_1 = \psi_1^*. \qquad [48]$$

From [43] the expression for g_1 at $y = 1$ is,

$$g_1 = -\psi_1^* / \psi_{0y}^* \qquad [49]$$

and substitution of [49] into [44] at $y = 1$ yields,

$$\psi_{1y} - \psi_{0yy}\psi_1^*/\psi_{0y}^* = \psi_{1y}^* - \psi_{0yy}^*\psi_1^*/\psi_{0y}^*. \qquad [50]$$

Similarly, the expression for g_1 can be substituted into [46] at $y = 1$ to obtain,

$$(\psi_{1yy}^* - \psi_{1xx}^* - \psi_{0yyy}^*\psi_1^*/\psi_{0y}^*)\alpha - \psi_{1yy} + \psi_{1xx} + \psi_{0yyy}\psi_1^*/\psi_{0y}^* = 0. \qquad [51]$$

In addition, [45] can also be rewritten,

$$-\frac{3}{2}\psi_{1xxy} - \frac{1}{2}\psi_{1yyy} + \psi_{0yy}\psi_{1xx}^*/\psi_{0y}^* - \frac{\Lambda^{-1}}{2}\psi_{1xxx}^*/\psi_{0y}^* =$$

$$\left(-\frac{3}{2}\psi_{1xxy}^* - \frac{1}{2}\psi_{1yyy}^* + \psi_{0yy}^*\psi_{1xx}^*/\psi_{0y}^*\right)\alpha \qquad [52]$$

after recognizing that at $y = 1$,

$$p_{1x} = \psi_{1xxy} + \psi_{1yyy} \qquad [53]$$

$$p_{1x}^* = (\psi_{1xxy}^* + \psi_{1yyy}^*)\alpha. \qquad [54]$$

Consequently, the periodicity condition of ψ_1 and ψ_1^* on x suggests one to write,

$$\psi_1 = e^{i\lambda x} F(y) \qquad [55]$$

$$\psi_1^* = e^{i\lambda x} E(y) \qquad [56]$$

where the real part is only considered in the solution, and i indicates the complex variable. Once [55] and [56] are substituted into [38]–[41], [47], [48] and [50]–[52] two ordinary differential equations result,

$$F_{yyyy} - 2\lambda^2 F_{yy} + \lambda^4 F = 0 \quad [57]$$

$$E_{yyyy} - 2\lambda^2 E_{yy} + \lambda^4 E = 0 \quad [58]$$

with the following boundary conditions:
at $y = 1 + m$,

$$F_y = i(6a(1+m) + 2b) \quad [59]$$

$$F = 0 \quad [60]$$

at $y = 1$,

$$F = E \quad [61]$$

$$F_y - E_y + [(6(a^* - a) - 2b)/(3a^* + c^*)]E = 0 \quad [62]$$

$$\frac{3}{2}\lambda^2 F_y - \frac{1}{2}F_{yyy} + \left(E_{yyy}/2 - \frac{3}{2}\lambda^2 E_y\right)\alpha +$$

$$+ \left[\lambda^2\left(6a^*\alpha - 6a - 2b + i\lambda\frac{\Lambda^{-1}}{2}\right)\Big/(3a^* + c^*)\right]E = 0 \quad [63]$$

$$E_{yy}\alpha + [\alpha\lambda^2 + 6(a - a^*\alpha)/(3a^* + c^*)]E - F_{yy} - \lambda^2 F = 0 \quad [64]$$

and at $y = 0$,

$$E = 0 \quad [65]$$

$$E_{yy} = 0. \quad [66]$$

Solutions to [57] and [58] are,

$$F(y) = (c_1 + c_2 y)\sinh(\lambda y) + (c_3 + c_4 y)\cosh(\lambda y) \quad [67]$$

$$E(y) = (c_1^* + c_2^* y)\sinh(\lambda y) + (c_3^* + c_4^* y)\cosh(\lambda y) \quad [68]$$

where eight complex constants must be evaluated with boundary conditions given by [59]–[66].
Since $E(y)$ is a complex function, one can also write,

$$E(y) = K(y) + iL(y) \quad [69]$$

Therefore, from [49], [56] and [69]

$$g_1(x) = \mathrm{Am}\sin(\lambda x + \gamma) \quad [70]$$

being the sinsoidal character of the interface a consequence of linearization.
The amplitude of the interfacial wave Am as well as the phase shift γ are expressed,

$$\mathrm{Am} = \sqrt{K^2(1) + L^2(1)}/(3a^* + c^*) \quad [71]$$

$$\gamma = \mathrm{arc\,tg}\left(-\frac{K(1)}{L(1)}\right) \qquad [72]$$

γ units are radians in this equation.

Since the resulting equations of the $0(\epsilon)$ problem are complex, a numerical algorithm was used to evaluate the complex coefficients in [67] and [68].

RESULTS AND DISCUSSION

Evaluations of the two phase flow in steady state through a sinusoidal channel show that there exists a phase-shift between the sinusoidal solid–fluid interface and the wavy shaped fluid–fluid interface. Since the above theoretical analysis is carried out for small Reynolds numbers as well as for a small ratio between the amplitude of variation of the channel wall and the average depth of the non-wetting phase, these constraints must be systematically assumed in the following discussion of results.

Figures 2–5 present the dependence of the phase shift γ and the amplitude Am of the fluid–fluid interface as functions of the geometrical parameter λ, for different values of the capillary number Λ, flow rate ratio q, viscosity ratio α and phase thickness ratio m.

In general the amplitude Am decreases with λ and tends asymptotically to zero when $\lambda \gg \epsilon$ (figures 2 and 3). This limit is expected, since for $l \ll 2\pi A^*/\epsilon$ the channel wall variations are so closed that the wetting fluid flows according to the minimum channel gap. On the other hand, the phase shift γ presents a peak whose magnitude depends on the values of physical and geometrical parameters (figures 4 and 5). One also observes that $\gamma \to 0$ as $\lambda \to 0$ because $A^* \ll l$, and the fluid varies, therefore, very slowly in the x-direction. It is appropriate to observe here that this variation in the flow field is generating the phase shift between the fixed solid–liquid and the liquid–liquid interfaces.

In order to analyze consistently further results, one has to recognize the set of independent variables (m, q, α, θ, Λ, λ) of which the phase-shift γ and the amplitude Am are functions. Therefore, one independent variable can be changed while the others are kept constant so that the effects of this change on Am and γ are evaluated. As a result of this analysis, a physical criterion concerning the mechanisms by which the two-phase flow destabilizes, can be suggested.

It is thus found that the phase-shift γ decreases with the capillary number Λ because of the effect of the surface tension on the interfacial deformation. Nevertheless, the capillary number does not affect the amplitude Am, which is, therefore, generated by bulk stresses.

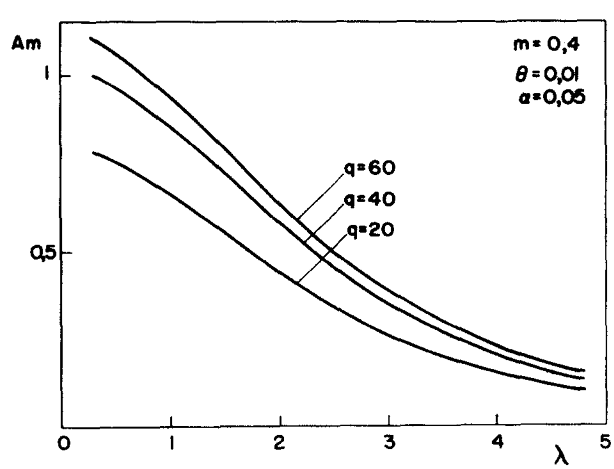

Figure 2. Amplitude of the fluid–fluid interface Am as a function of the geometrical parameter λ, for different values of the flow ratio q, and $m = 0.4$; $\theta = 0.01$; $\alpha = 0.05$.

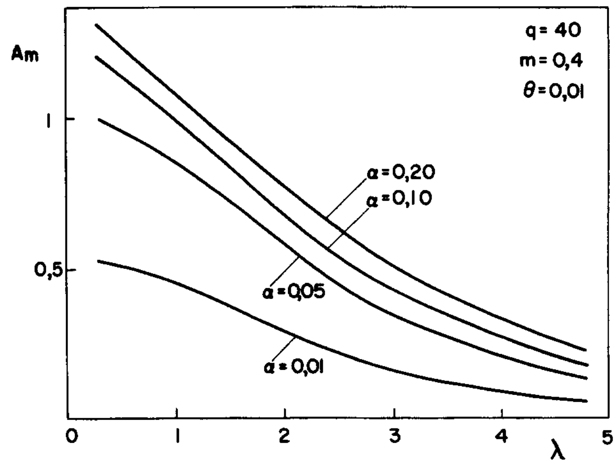

Figure 3. Amplitude of the fluid–fluid interface Am as a function of the geometrical parameter λ, for different values of the viscosity ratio α, and $q = 40$; $m = 0.4$; $\theta = 0.01$.

Other significant results are:

(a) As the flow rate ratio q is increased the amplitude Am increases and the phase-shift γ decreases, this effect being significant at lower values of λ. Thus, increasing the flow rate of the non-wetting phase or decreasing the flow rate of the wetting phase the amplitude of the fluid–fluid interfacial wave increases.

(b) As the viscosity ratio α is increased the amplitude Am increases and the phase-shift γ decreases, being this effect again significant at lower values of λ. Consequently, a great difference of viscosity between phases generates an appreciable wave at the fluid–fluid interface, which is almost in phase with the solid–fluid interface.

(c) As the density ratio θ is increased the amplitude Am and the phase-shift γ do not change at all. This is consistent with the theoretical analysis, since the results are only valid for Reynolds numbers of the order of ϵ, and inertial effects are, therefore, of order ϵ^2.

(d) As the thickness ratio m is increased at low values of λ, the amplitude Am increases and the phase-shift γ increases. Although this is exactly the opposite for Am at large values of λ,

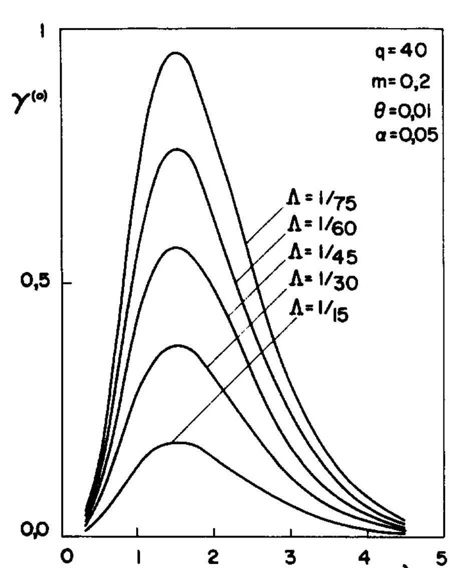

Figure 4. Phase-shift γ (in degrees) as a function of the geometrical parameter λ, for different values of the capillary number, and $q = 40$; $m = 0.2$, $\theta = 0.01$, $\alpha = 0.05$.

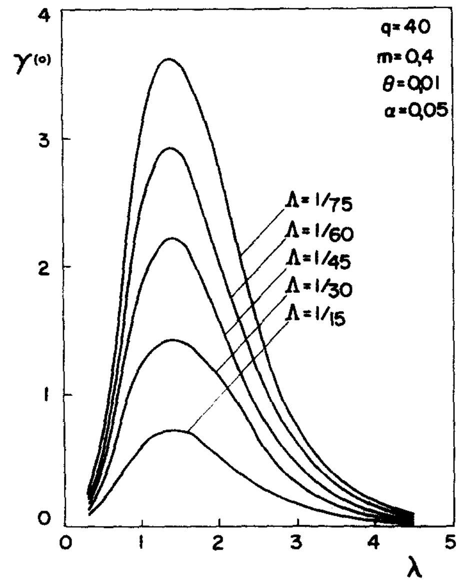

Figure 5. Phase-shift γ (in degrees) as a function of the geometrical parameter λ, for different values of the capillary number, and $q = 40$; $m = 0.4$; $\theta = 0.01$; $\alpha = 0.05$.

Table 1.

m = 0.2 ; α = 0.01 ; θ = 0.005 ; λ = 0.9		
q	$\Delta p + O(\varepsilon^2)$	N
40.	$- 0.72\ 10$	$0.3965\ 10$
60.	$- 0.11\ 10^2$	$0.6231\ 10$
80.	$- 0.15\ 10^2$	$0.8497\ 10$

q = 20. ; m = 0.2 ; θ = 0.005 ; λ = 0.9		
α	$\Delta p + O(\varepsilon^2)$	N
0.5	$- 0.13\ 10^3$	$0.7123\ 10^2$
0.7	$- 0.17\ 10^3$	$0.9354\ 10^2$
0.9	$- 0.20\ 10^3$	$0.1133\ 10^3$

q = 20 ; α = 0.01 ; θ = 0.005 ; λ = 0.9		
m	$\Delta p + O(\varepsilon^2)$	N
0.2	$- 0.31\ 10$	$0.1699\ 10$
0.3	$- 0.34\ 10$	$0.2781\ 10$
0.4	$- 0.34\ 10$	$0.4882\ 10$
0.5	$- 0.32\ 10$	$0.1203\ 10^3$

this inversion of the dependence of Am with λ and m is not physically important, because $Am \to 0$ as λ becomes large ($\lambda \gg \epsilon$), for any value of m.

The pressure drop measured along the wave length l of the cell increases as the flow rate and the viscosity ratios are increased, as expected. Nevertheless, it may present a maximum value with variations of the phase-thickness ratios (see table 1).

In all these calculations it was found that the integration [53] and [54] yields $\Delta p_1 = 0$.

A complete stability analysis of this flow problem has not been performed in the literature yet, and one expects to face a difficult problem. From a physical point of view, it is possible, however, to infer that the increments of the fluid–fluid interfacial amplitude Am and the phase-shift γ tend to destabilize the two-phase flow in a given channel. Consequently, instabilities are expected at large values of α, m and q, at intermediate values of λ and at small values of the capillary number Λ. Since this conclusion is valid for small Reynolds numbers of the order ϵ, a change in the flow regime without turbulence can take place; for example, one phase might be displaced by the other, so that, both phases wet the sinusoidal wall of the channel.

Wang (1981) analyzed the flow of an incompressible fluid on a sinusoidal incline, and the interfacial phase-shift and amplitude were also evaluated. Nevertheless, his results do not include the effect of α, θ, m, and q because the second phase is neglected. Although some qualitative comparisons with Wang's results show a good agreement with the above results, one should note that the kinematic boundary condition at the interface when one phase is neglected is substantially different from that used in the present work [13].

CONCLUSIONS

The two-phase flow of two immiscible fluids through a sinusoidal channel is solved for small values of the Reynolds number and small ratio between the amplitude of variation of the channel wall and the thickness of the non-wetting phase. The flow can be characterized under these constraints with six parameters: viscosity, density, phase-thickness and flow rate ratios, the capillary number and a geometrical ratio between one of the phase thicknesses and the wave length of the sinusoidal wall of the channel.

The wavy shaped fluid–fluid interface presents a phase-shift in relation to the sinusoidal solid–fluid interface and also a characteristic amplitude of the wave. Therefore, the dependence of the interfacial amplitude and phase-shift on the six physical and geometrical parameters is established as well as a physical mechanism of flow destabilization is suggested.

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NOTATION

- A average thickness of the wetting phase
- A^* average thickness of the non-wetting phase
- Am amplitude of the interfacial wave
- B magnitude of variation in the cross area of flow
- E complex valued function of y
- F complex valued function of y
- $f(x)$ real valued function of x
- g gravity acceleration
- $2H$ interface curvature
- i complex variable.
- K real part of E.

L imaginary part of E.
l wave length of the periodic solid wall.
N $gA^{*3}/\nu Q$
m A/A^*
P pressure
p $P/(\mu Q/A^{*2})$
Q axial flow rate per unit length
q Q^*/Q
Re Q/ν, Reynolds number
$\mathbf{T}$ stress tensor
u x-direction velocity
v y-direction velocity
x Cartesian coordinate
y Cartesian coordinate
Y_0 position of the solid wall
Y_1 position of the fluid–fluid interface

Greek symbols

α μ^*/μ
γ phase shift between fluid–solid and fluid–fluid interfaces
ϵ B/A^*
θ ρ^*/ρ
Λ $\mu Q/\sigma A^*$, capillary number
λ $2\pi A^*/l$
μ viscosity
ν kinematic viscosity
ρ density
σ surface tension
ψ stream line function

Superscripts

* indicates non-wetting phase.

Subscripts

0 indicates terms of $0(1)$
1 indicates terms of $0(\epsilon)$
x x-direction derivate
y y-direction derivate

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